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Linear/Nonlinear Supersonic Panel Flutter in a High-Temperature Field

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An analysis of the flutter and postflutter behavior of infinitely long flat panels in a supersonic/hypersonic flowfield exposed to a high-temperature field is presented. In the approach to the problem, the thermal degradation of thermoelastic characteristics of the material is considered. A third-order piston theory aerodynamic model in conjunction with the von Kármán nonlinear plate theory is used to obtain the pertinent aerothermoelastic governing equations. The implications of temperature, thermal degradation, and of structural and aerodynamic nonlinearities on the character of the flutter instability boundary are analyzed. As a byproduct, the implications of the temperature on the linearized flutter instability of the system are discussed. The behavior of the structural system in the vicinity of the flutter boundary is studied via the use of an encompassing methodology based on the Lyapunov First Quantity. Numerical illustrations, supplying pertinent information on the implications of the temperature field and of the thermal degradation are presented, and pertinent conclusions are outlined.

Nomenclature

a_∞	=	speed of sound of the undisturbed flow, m/s
B^T	=	thermal load
b	=	panel width, m
D	=	flexural panel stiffness, $Eh^3/12(1-\mu^2)$, N · m
E, E_0, E_1, e	=	elastic moduli, Eq. (3)
h	=	panel thickness, m
L	=	Lyapunov First Quantity
M_∞, q_∞	=	undisturbed flight Mach number, $\rho_\infty U_\infty^2/2$ and dynamic pressure, kg/m s^2
M_F, U_F	=	Mach flutter and flutter speed, m/s
m, n, c	=	roots of the characteristic equation, Eqs. (32)
N_x, N_y, N_{xy}	=	membrane stress resultants, measured per unit length of the panel, N/m
p, q, r, s	=	coefficients of the characteristic equation, Eqs. (32)
p_∞	=	freestream pressure of the undisturbed flow, Pa
$\bar{p}$	=	dimensionless counterpart of q_a, $(q_a/\rho_\infty U_\infty^2)$
$\wp$	=	residual function
q_a	=	pressure difference $p - p_\infty$, Pa
q_n	=	generalized coordinate, $n = 1, 2, \ldots$
$\Re$	=	boundary of the region of stability
$T(y, z)$	=	temperature increment from a stress-free reference temperature T_0, °C
$\mathcal{T}$	=	temperature amplitude, °C
t	=	time, s
$\bar{t}$	=	dimensionless counterpart of t, (t/τ)
U_∞	=	air speed of the undisturbed flow, m/s
u, v, w	=	displacements in the x, y, and z, directions, m
v_z	=	downwash velocity normal to the cylindrical panel, m/s
$\bar{w}$	=	dimensionless transverse deflection, (w/h)
$x, y, z; \bar{y}$	=	Cartesian coordinates; dimensionless coordinate, y/b, respectively
$\alpha_p, \alpha_0, \alpha_1; \alpha$	=	coefficients of thermal expansion, Eq. (3)
γ	=	Glauert's aerodynamic correction factor
Δ	=	Laplace operator
Δ_y	=	end shortening in the y direction
δ_{ij}	=	tracers identifying the linear and nonlinear aerodynamic terms, $i = 1, 2, 3$ and $j = y, t$
$\varepsilon_x, \varepsilon_y, \varepsilon_{xy}$	=	tangential strains
κ	=	polytropic gas constant
λ_F	=	normalized dynamic pressure, $(2q_\infty b^3/M_\infty D_0)$
μ	=	Poisson's ratio
ρ_p	=	panel mass density, kg/m^3
ρ_∞	=	air density of the undisturbed flow, kg/m^3
$\bar{\rho}$	=	dimensionless counterpart of ρ_p $(\rho_\infty b/\rho_p h)$
$\sigma_x, \sigma_y, \sigma_{xy}$	=	in-plane stress components, N/m^2
ϕ	=	Airy's function
ω	=	frequency, rad/s

Subscripts

$, y; , xy$	=	$\partial(\cdot)/\partial y$; $\partial^2(\cdot)/\partial x \partial y$
$, t; , tt$	=	$\partial(\cdot)/\partial t$; $\partial^2(\cdot)/\partial t^2$
$+; -$	=	quantity evaluated on the upper $(z > 0)$ and lower $(z < 0)$ surfaces of the panel

Introduction

AERODYNAMIC thermal effects induced at high-speed flight may produce deformations, stresses, and changes in material properties that can dramatically affect the aeroelastic behavior of flight/space vehicles. In this sense, the structural panels of supersonic/hypersonic flight vehicles can experience, among others, the thermal flutter instability generated by the combined influence of the thermal field, unsteady aerodynamic loads, elasticity of structures, and the dynamic effects.[1]

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As a result of structural nonlinearities inherently present in the structure, in the postflutter regime, the panel amplitudes remain bounded in a limit-cycle motion.[2–4] In fact, in the post-critical range, when, in addition to the bending stiffness, the stretching also contributes to the overall stiffness of the panel, a stable limit cycle is expected to occur.[5] In such conditions, due to high oscillation frequencies, fatigue failure of the panel may endanger the wing structure. However, in the hypersonic flight speed range, when the aerodynamic nonlinearities become prevalent, a behavior opposite in character to that obtained in the absence of aerodynamic nonlinearities can emerge, in the sense that at high flight Mach numbers the flutter boundary can become catastrophic.[6–8] Because, due to the presence of the various nonlinearities, that is, geometrical and aerodynamic, and of the thermal field,[6,9] the character of the flutter boundary can be benign or catastrophic, the analysis of the postflutter behavior becomes obviously impotant. In addition, the mechanical properties of constituent materials of structural components of supersonic flight vehicles can be dramatically affected by the temperature, in the sense of a thermal degradation of the material properties. This thermal degradation of their properties can modify in a detrimental way the flutter boundary and the character of the flutter boundary. Note that, in spite of the great importance of the problem, few investigations on panel flutter that include the thermal effects have been considered so far.[5,10–13]

In this paper, the study of the flutter and postflutter behavior of supersonic panels in the presence of a high-temperature environment, of the thermal degradation of the material properties, and of consideration of structural and aerodynamic nonlinearities is addressed.

Aerothermoelastic Governing Equations

To derive the aerothermoelastic governing equations, the geometrically nonlinear theory of infinitely long flat panels is considered. In this context, the Kirchhoff plate model in conjunction with the von Kármán nonlinear strain–displacement approximation is adopted. In addition, the effect of a thermal field is included. In the case of a flat panel, the equation of motion expressed in terms of the transverse deflection w and the Airy's function ϕ is

$$(D/h)\nabla^4 w = w_{,xx}\phi_{,yy} + w_{,yy}\phi_{,xx} - 2w_{,xy}\phi_{,xy} + q_a/h - \rho_p w_{,tt} + \Delta B^T \tag{1}$$

Herein, $D[\equiv h^3 E/12(1-\mu^2)]$ is the flexural stiffness, where E is Young's modulus, q_a is the transversal pressure difference, ϕ is Airy's potential function, $\nabla^2(\cdot) \equiv \Delta(\cdot)[\equiv(\cdot)_{,xx} + (\cdot)_{,yy}]$ denotes the Laplace operator, and B^T is the thermal load defined as

$$B^T = \frac{E\alpha_p}{1-\mu}\int_{-h/2}^{h/2} T(y,z)z\,\mathrm{d}z \tag{2}$$

In Eq. (2), $T(y,z)$ denotes the temperature increment from a stress-free reference temperature T_0. In addition, the material properties of the panel, E and α_p, influenced by the thermal field are expressed as

$$E = E_0 + E_1 T = E_0(1+eT), \qquad \alpha_p = \alpha_0 + \alpha_1 T = \alpha_0(1+\alpha T) \tag{3a}$$

where

$$e(\equiv E_1/E_0) < 0, \qquad \alpha(\equiv \alpha_1/\alpha_0) > 0 \tag{3b}$$

are the coefficients associated with the thermal degradation. For $e = \alpha = 0$, the thermal degradation is immaterial. As assumed throughout the specialized literature,[14] and also considered here, Poisson's ratio is marginally affected by the temperature.

A linear temperature distribution T throughout the panel thickness is considered,

$$T(y,z) = \overset{0}{T}(y) + z\overset{1}{T}(y) \tag{4}$$

Note that this temperature distribution was obtained via an exact analysis by Bolotin.[9] As a result of the temperature dependence of the thermoelastic material properties and of the spatially distributed temperature field, the thermoelastic coefficients of the material become functions of the space variables in the sense of $E \Rightarrow E(y)$ and $\alpha_p \Rightarrow \alpha_p(y)$. This implies that an induced nonhomogeneity of the structural panel is resulting.

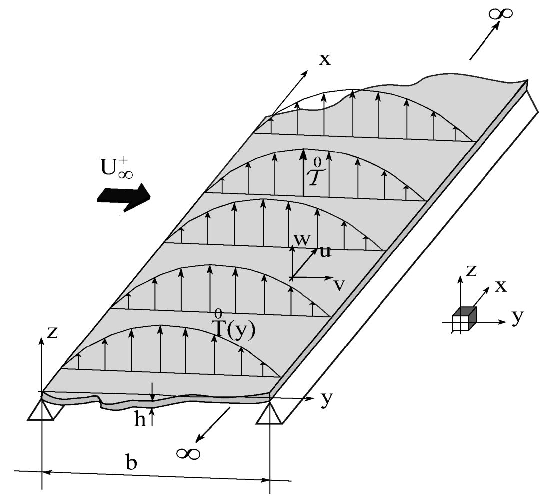

Fig. 1 Flat panel geometry.

Equation (1) is supplemented by the compatibility equation,

$$\frac{1}{E(y)}\nabla^4\phi = w_{,xy}^2 - w_{,xx}w_{,yy} - \alpha_p(y)\Delta\int_{-h/2}^{h/2} T(y,z)\,\mathrm{d}z \tag{5}$$

For infinitely long flat panels in the x direction, in addition to the proper condition $(\cdot)_{,x} \Rightarrow 0$, the displacements quantities should fulfill the conditions

$$u = 0, \qquad v \Rightarrow v(y,t), \qquad w \Rightarrow w(y,t) \tag{6}$$

where u, v, and w are the displacement components in the x, y, and z, directions, respectively (Fig. 1).

Under the conditions stipulated by Eq. (6), Eq. (5) is identically fulfilled, and as a result, it becomes immaterial. The components of the strain tensor at every point of the panel, ε_x, ε_y, and ε_{xy} are related to the components of the stress tensor σ_x, σ_y, and σ_{xy} as follows:

$$\begin{aligned}\varepsilon_x &= [1/E(y)](\sigma_x - \mu\sigma_y) + \alpha_p(y)T\\ \varepsilon_y &= [1/E(y)](\sigma_y - \mu\sigma_x) + \alpha_p(y)T\\ \varepsilon_{xy} &= [1/E(y)](1+\mu)\sigma_{xy}\end{aligned} \tag{7}$$

The inverted counterparts of Eqs. (7) are

$$\sigma_x = N_x/h = \phi_{,yy} = [E(y)/(1-\mu^2)]\big[\varepsilon_x + \mu\varepsilon_y - (1+\mu)\alpha_p(y)\overset{0}{T}\big] = 0 \tag{8}$$

$$\sigma_y = N_y/h = \phi_{,xx} = [E(y)/(1-\mu^2)]\big[\varepsilon_y + \mu\varepsilon_x - (1+\mu)\alpha_p(y)\overset{0}{T}\big] \equiv \sigma \tag{9}$$

$$\sigma_{xy} = N_{xy}/h = -\phi_{,xy} = [E(y)/(1+\mu)]\varepsilon_{xy} = 0 \tag{10}$$

where N_x, N_y, and N_{xy} are the membrane stress resultants. Equations (8–10) imply that the tangential stresses act only in the y direction. Physically, this stress is generated by the constraint of the panel with the members of the airframe. In light of the earlier stipulated assumptions, Eq. (1) becomes

$$\frac{D}{h}w_{,yyyy} = w_{,yy}\sigma_y + \frac{q_a}{h} - \rho_p w_{,tt} + \lambda\Delta\int_{-h/2}^{h/2} Tz\,\mathrm{d}z \tag{11}$$

where $\lambda = E\alpha_p/(1-\mu)$. To evaluate the tangential stress component σ_y, one expresses the average end shortening Δ_y in the form[6,7]

$$\Delta_y(t) = -\frac{1}{b}\int_0^b v_{,y}\,\mathrm{d}y \tag{12}$$

and in the case of immovable edges $y = 0, b$, considered in this paper,

$$\int_0^b v_{,y}\,\mathrm{d}y = 0 \Longrightarrow \Delta_y = 0 \tag{13}$$

On the other hand, based on the expressions of the strain–displacement relationship expressed in the Lagrangian description in conjunction with the adoption of the von Kármán assumption, the strain in the y direction, ε_y, becomes

$$\varepsilon_y = v_{,y} + \tfrac{1}{2}(w_{,y})^2 \tag{14}$$

Replacing Eq. (14) in Eq. (9), in conjunction with the fact that $\varepsilon_x = 0$, one obtains

$$\sigma_y = [E(y)/(1-\mu^2)]\left[v_{,y} + \tfrac{1}{2}(w_{,y})^2 - (1+\mu)\alpha_p(y)\overset{0}{T}\right] \tag{15}$$

Solving now Eq. (15) for $v_{,y}$, one obtains

$$v_{,y} = (1-\mu^2)[\sigma_y/E(y)] - \tfrac{1}{2}(w_{,y})^2 + (1+\mu)\alpha_p(y)\overset{0}{T} \tag{16}$$

Applying the operator

$$\frac{1}{b}\int_0^b (\cdot)\,\mathrm{d}y$$

to Eq. (16), considered in conjunction with Eqs. (12) and (13), yields

$$\sigma_y = \left[1\bigg/(1-\mu^2)\int_0^b E(y)^{-1}\,\mathrm{d}y\right] \times \left[\frac{1}{2}\int_0^b (w_{,y})^2\,\mathrm{d}y - (1+\mu)\int_0^b \alpha_p(y)\overset{0}{T}\,\mathrm{d}y\right] \tag{17}$$

Substitution of Eq. (17) in Eq. (11) results in

$$Dw_{,yyyy} - \left[h\bigg/(1-\mu^2)\int_0^b E(y)^{-1}\,\mathrm{d}y\right]\left[\frac{1}{2}\int_0^b (w_{,y})^2\,\mathrm{d}y - (1+\mu)\int_0^b \alpha_p(y)\overset{0}{T}\,\mathrm{d}y\right]w_{,yy} - q_a + \rho_p h w_{,tt} - \lambda\Delta\int_{-h/2}^{h/2} T(y,z)z\,\mathrm{d}z = 0 \tag{18}$$

Equation (18) represents the geometrically nonlinear aerothermoelastic governing equation of infinitely long flat panels with immovable edges whose constituent material features thermal degradation. Notice that, in the absence of the temperature field, Eq. (18) reduces to a form extensively used in nonlinear panel flutter investigations, for example, Refs. 2–4 and 15. For the problem at hand, the term

$$\lambda\Delta\int_{-h/2}^{h/2} T(y,z)z\,\mathrm{d}z$$

in conjunction with Eq. (4) yields the thermal moment given by $\lambda h^3 \overset{1}{T}_{,yy}/12$.

A membrane temperature distribution $\overset{0}{T}$, implying $\overset{1}{T} = 0$, will be considered. This temperature distribution can correspond to the steady-state flight regime of a high-speed aerospace vehicle. Such a representation of the temperature field is adopted here to reduce the problem to an eigenvalue one. Specifically, $\overset{0}{T}(y)$ is expressed as (Fig. 1)

$$\overset{0}{T}(y) = \overset{0}{T}\sin(\pi y/b) \tag{19}$$

where $\overset{0}{T}$ is the temperature amplitude at $y = b/2$.

Piston Theory Aerodynamics

To study the nonlinear panel flutter, in addition to the inclusion of geometrical nonlinearities, a nonlinear piston theory aerodynamics (PTA) model is used. PTA is a popular modeling technique for supersonic and hypersonic aeroelastic analyses.[16] Consistent with it, the unsteady aerodynamic pressure can be expressed as

$$p(y,t) = p_\infty\{1 + [(\kappa-1)/2](v_z/a_\infty)\}^{2\kappa/(\kappa-1)} \tag{20}$$

where the downwash velocity v_z normal to the panel and the undisturbed speed of sound a_∞ are expressed as

$$v_z = -(w_{,t} + U_\infty w_{,y}), \qquad a_\infty^2 = \kappa p_\infty/\rho_\infty \tag{21}$$

Here p_∞, ρ_∞, and U_∞ are the pressure, air density, and the air speed of the undisturbed flow, respectively. Retaining, in the binomial expansions of Eq. (20), the terms up to and including $(v_z/a_\infty)^3$ yields the pressure formula for the PTA in the third-order approximation[8]

$$p/p_\infty = 1 + \kappa(v_z/a_\infty)\gamma + [\kappa(\kappa+1)/4][(v_z/a_\infty)\gamma]^2 + [\kappa(\kappa+1)/12][(v_z/a_\infty)\gamma]^3 \tag{22}$$

In Eq. (22), the aerodynamic correction factor $\gamma = M_\infty/\sqrt{(M_\infty^2 - 1)}$ enables one to extend the validity of the PTA to the entire low supersonic/hypersonic flight speed regime. For more details regarding the PTA and its applicability see, for example, Refs. 7 and 16. Note that PTA provides results in excellent agreement with those based on the Euler solution and the CFL3D code[18] and with the exact unsteady supersonic aerodynamics theory.[19]

Consider the flow only on the upper surface of the panel $U_\infty^+ \equiv U_\infty$ and $M_\infty = U_\infty^+/a_\infty$, that is, consider $U_\infty^- = 0$ and $p^- = p_\infty$; from Eqs. (20–22), the aerodynamic pressure difference can be expressed as

$$q_a = p - p_\infty = \delta p|_{\mathrm{PTA}} = -(2q_\infty/M_\infty)\gamma\Big\{(1/U_\infty)w_{,t} + w_{,y} + [(1+\kappa)/4]\gamma M_\infty \times [(1/U_\infty)w_{,t} + w_{,y}]^2 + [(1+\kappa)/12]\gamma^2 M_\infty^2[(1/U_\infty)w_{,t} + w_{,y}]^3\Big\} \tag{23}$$

$M_\infty = U_\infty/a_\infty$ is the undisturbed flight Mach number, whereas $q_\infty = \rho_\infty U_\infty^2/2$ is the undisturbed dynamic pressure. Using the dimensionless parameters presented in Appendix A, the governing equation (18) reduces to

$$\left(1 + \delta_e e\overset{0}{\bar{T}}\hat{T}\right)\bar{w}_{,\overline{yyyy}} - \left[1\bigg/\int_0^1\left(1 + \delta_e e\overset{0}{\bar{T}}\hat{T}\right)^{-1}\mathrm{d}\bar{y}\right] \times \left[6\int_0^1 (\bar{w}_{,\bar{y}})^2\,\mathrm{d}\bar{y} - \frac{1}{(1-\mu)}\int_0^1\left(1 + \delta_\alpha\alpha\overset{0}{\bar{T}}\hat{T}\right)\overset{0}{\bar{T}}\,\mathrm{d}\bar{y}\right]w_{,\overline{yy}} - \bar{q}_a + \bar{w}_{,\overline{tt}} = 0 \tag{24}$$

where the aerodynamic load is cast as

$$\bar{q}_a = \lambda\gamma(b/h)\Big\{\left[\sqrt{\bar{\rho}/\lambda M_\infty}(h/b)\bar{w}_{,\bar{t}} + (h/b)\bar{w}_{,\bar{y}}\right] + [(1+\kappa)/4]\gamma M_\infty\left[\sqrt{\bar{\rho}/\lambda M_\infty}(h/b)\bar{w}_{,\bar{t}} + (h/b)\bar{w}_{,\bar{y}}\right]^2 + [(1+\kappa)/12]\gamma^2 M_\infty^2\left[\sqrt{\bar{\rho}/\lambda M_\infty}(h/b)\bar{w}_{,\bar{t}} + (h/b)\bar{w}_{,\bar{y}}\right]^3\Big\} \tag{25}$$

The tracers δ_e and δ_α in Eq. (24) identify the terms associated with the thermal degradation of the elastic modulus and of the coefficient of thermal expansion, respectively. These take the values 1 or 0 depending on whether the respective effect is included or discarded.

Solution Methodology Based on Galerkin and Lyapunov First Quantity

Galerkin Based Approach

For the solution of the nonlinear flutter problem, the harmonic balance technique, the direct numerical integration technique, and the perturbation methods can be used. More recently, limit-cycle amplitude panel flutter using the finite element method was studied.[15,20] In the present approach, an analytical formulation based on the Lyapunov First Quantity in conjunction with Galerkin's method enables one to evaluate the character of the flutter boundary of the panel with temperature-dependent thermoelastic properties. This methodology, presented in Refs. 7 and 21 and adopted in Refs. 8 and 22, has been expanded here to include the temperature and the thermal degradation effects of the structure. Notice that the same methodology was used quite recently to address the problem of Hopf bifurcation of aeroelastic systems with a time delay in the controls.[23]

Note that Eq. (24) has to be solved with given boundary conditions. For the case of simply supported panels, one expresses $\bar{w}(\bar{y}, \bar{t})$ as

$$\bar{w}(\bar{y}, \bar{t}) = \sum_{n=1}^{\infty} q_n(\bar{t}) f_n(\bar{y}) \tag{26}$$

When $f_n(\bar{y}) = \sin(\lambda_n \bar{y})$ and $\lambda_n = n\pi$, $n = 1, 2, \ldots$, are considered as modal functions, all of the boundary conditions that concern the transverse deflection $\bar{w}$ are fulfilled. The generalized coordinates $q_n(\bar{t})$ are time-dependent functions, whereas to address the free vibration problem and to solve the linearized problem resulting in the evaluation of the flutter characteristics, $q_n(\bar{t})$ can be expressed as

$$q_n(\bar{t}) = \sum_{n=1}^{k} a_n \exp(i\bar{\omega}_n \bar{t}) \tag{27}$$

where i is the imaginary unit. The natural frequencies of the panel in the presence of the temperature field can be obtained numerically via Galerkin's method. Replacing Eq. (26) in Eq. (24), the obtained residual denoted as $\wp(\bar{y}, \bar{t})$ is minimized in Galerkin's sense:

$$\int_0^1 \wp(\bar{y}, \bar{t}) f_m(\bar{y})\, \mathrm{d}\bar{y} = 0 \tag{28}$$

As a result of Eq. (28), a set of nonlinear, simultaneous differential equations with respect to the coefficients of the series in Eq. (26) is obtained. In condensed form the system can be written as

$$\frac{\mathrm{d}^2 q_m}{\mathrm{d}\bar{t}^2} + g\frac{\mathrm{d}q_m}{\mathrm{d}\bar{t}} + F_m(q_n, M_\infty) = 0 \qquad n, m = 1, 2, 3, \ldots \tag{29}$$

The term $g\, \mathrm{d}q_m/\mathrm{d}\bar{t}$ associated with the structural damping has been discarded. Although structural damping can be included in the analysis, the present simulations involve only aerodynamic damping. In such a context, more conservative estimates of the flutter speed are expected to occur. The functions $F_m(q_n, M_\infty)$ can be represented as

$$F_m(q_n, M_\infty) = F_m^{(l)}(q_n, M_\infty) + F_m^{(nl)}(q_n, M_\infty) \tag{30}$$

where $F_m^{(l)}(q_n, M_\infty)$ are linear functions that contain also the thermal load, whereas $F_m^{(nl)}(q_n, M_\infty)$ are functions that include the aerodynamic and structural nonlinearities and the thermal degradation.

Stability in the Vicinity of the Critical Flutter Boundary via Lyapunov First Quantity

From the mathematical point of view, the issue of the character of the flutter boundary, that is, benign or catastrophic, can be revealed via determination of the nature of the Hopf bifurcation, that is, supercritical or subcritical, respectively, as featured by the nonlinear aeroelastic system. The conditions of catastrophic/benign character of the flutter instability boundary are obtained via the use of the Lyapunov first quantity (LFQ) (see Ref. 8). This quantity will be evaluated next.

To this end, the system of governing equations (29) is converted to a system of four differential equations in state-space form expressed generically as

$$\frac{\mathrm{d}x_j}{\mathrm{d}\bar{t}} = \sum_{m=1}^{n} a_m^{(j)} x_m + P_j(x_1, x_2, \ldots, x_n), \qquad j = \overline{1, 4} \tag{31}$$

For the present case, the functions $P_j(x_1, x_2, \ldots, x_n)$ include both the structural and aerodynamic nonlinear terms as well as the temperature-dependent terms. In the absence of thermal terms, its expression can be found in Ref. 8. Equation (31) can be presented in a form that can then be used toward the evaluation of the LFQ, $L(M_F)$. Considering the solution of the linearized counterpart of Eq. (31) under the form $x_j = A_j \exp(\omega t)$, one obtains the characteristic equation

$$\omega^4 + p\omega^3 + q\omega^2 + r\omega + s = 0 \tag{32}$$

As a reminder, for steady motion, the equilibrium is stable in Lyapunov's sense if the real parts of all of the roots of the characteristic equation are negative. Such an analysis can be done by applying Routh–Hurwitz's criterion. On this basis, the stability conditions reduce to $p > 0$, $q > 0$, $r > 0$, $s > 0$, and $\Re = pqr - sp^2 - r^2 > 0$. The roots of the characteristic equation on the critical flutter boundary, $\Re = 0$, are given by:

$$\omega_{1,2} = \pm ic, \qquad \omega_{3,4} = -m \pm in \tag{33}$$

and

$$c^2 = r/p, \qquad m = p/2, \qquad n^2 = sp/r - p^2/4 \tag{34}$$

with $n > 0$. For sufficiently small values of the speed, all of the roots of the characteristic equation are in the left half-plane of the complex variable, and the zero solution of the system is asymptotically stable. In addition, on the same boundary, the value $M_\infty = M_F$, where two roots of the characteristic equation are purely imaginary and the remaining two are complex conjugate and remain in the left half-plane of the complex variable, is critical and corresponds to the critical flutter velocity M_F. These conditions, expressed in full in Eq. (33), fulfill the Hopf bifurcation theorem (see Ref. 24).

Then, to identify the benign and catastrophic portions of the stability boundary (or in the terminology of the Hopf bifurcation, the supercritical from the subcritical ones), it is necessary to solve the problem of stability for the system of equations in state-space form in the critical case of a pair of pure imaginary roots.

Following the method developed by Bautin,[21] the benign or catastrophic portions of the boundary of the flutter boundary, that is, stable limit-cycle oscillation (LCO) and unstable LCO, respectively, can be determined via determination of the sign of the LFQ. The procedure to identify the LFQ is detailed in the Refs. 7 and 8.

The flutter critical boundary is benign that is, yields stable LCO, or is catastrophic, yielding unstable LCO, if the inequalities

$$L(M_F) < 0, \qquad L(M_F) > 0 \tag{35}$$

are fulfilled, respectively. As a general comment, the LFQ contains terms related to the structural and aerodynamic nonlinearities and the effect of the thermal load and thermal degradation. As it will be shown from the numerical simulations, these effects affect significantly the character of the flutter boundary. In the region of the benign flutter boundary, one can exceed the flutter critical speed M_F without a catastrophic failure of the panel. In this case, the amplitude of the transverse deflection remains limited. Conversely, in the region of catastrophic flutter boundary, an explosive type of flutter can occur.

Results and Discussion

Unless otherwise stated, the numerical simulations consider a Ti-8 Mn panel,[14] whose mechanical properties and geometric parameters are $E_0 = 10.3 \times 10^{10}$ Pa, $\alpha_0 = 4.7 \times 10^{-6}\, 1/^\circ\mathrm{C}$, $\mu = 0.3$, $\rho_p = 4520$ kg/m^3, $h/b = 0.005$, $\omega_1 \approx 10$ rad/s, $e = -0.0005$, and

$\alpha = 0.000389$. In addition, the flowfield characteristics are as follows: $\rho_\infty = 1.225$ kg/m^3, $\kappa = 1.4$, $\gamma = 1$, and $a_\infty = 340.3$ m/s. For this test case, the flutter speed is $U_F = 730$ m/s at $M_{\text{flight}} = 2$. As in Ref. 25, note that, in the case of immovable edges, due to the induced compressive stresses as a result of the edge constraints, a decrease of the thermomechanical buckling loads is experienced. Consequently, $\overset{0}{\mathcal{T}}$ has been prescribed in the subcritical buckling range, that is $\overset{0}{\mathcal{T}} \in [0, 10]$.

First, in Fig. 2 the first three eigenfrequencies as a function of the temperature amplitude are shown. These are obtained from Eq. (24) when the nonlinear terms are discarded. It clearly appears that with the increase of the temperature amplitude a decrease of the eigenfrequencies is experienced.

Figure 3 highlights the influence, on the first eigenfrequency, of the temperature in conjunction with the thermal degradation of thermomechanical properties of the material of the panel. It appears that, for small values of the temperature distribution amplitude $\overset{0}{\mathcal{T}}$, the thermal degradation of the elastic modulus has the strongest influence toward the decay of eigenfrequencies, whereas for larger values of $\overset{0}{\mathcal{T}}$, the thermal degradation of the thermal expansion coefficient becomes prevalent. As shown in Fig. 4, with the increase of the amplitude of the thermal field the flutter speed decreases. In Fig. 5 the influence of the panel thickness ratio on the flutter speed with/without thermal field (and in the absence of thermal degradation) is presented. It clearly appears that for larger panel thickness ratios the effect of the thermal field is more prominent in the detrimental sense than for thinner panels that are less sensitive to this effects. From Fig. 6, it also appears that, although an increase of the flutter speed is obtained by increasing the supersonic flight Mach number, the effect of the thermal degradation yields a reduction of the flutter speed within the entire supersonic/hypersonic flight speed range. It is also evident that, when considering the influence of the thermal degradation on the elastic modulus, that is, when $\delta_e = 1$ and $\delta_\alpha = 0$, lower values of the flutter speed are obtained as compared to those corresponding to the case when the coefficient of thermal expansion is affected only, that is, when $\delta_e = 0$ and $\delta_\alpha = 1$. A severe reduction of the flutter speed is noted when the degradation of both E and α is considered. Figures 7 and 8 depict the variation of the LFQ vs Mach flight. In Figs. 7 and 8, the effects of the structural and aerodynamic nonlinearities considered in conjunction with that of the temperature and the thermal degradation on the character of the flutter boundary are emphasized. The intersections of the curves $L(M_F)$ with the plane $L = 0$ separate the parts of the flutter boundary that are benign [for $L(M_F) < 0$] from the catastrophic ones, for which the opposite relationship $L(M_F) > 0$ is fulfilled.

For the panels whose materials do not experience the thermal degradation, Fig. 7 highlights the effect played by the thermal field on the character of the flutter boundary. As it can be seen, with

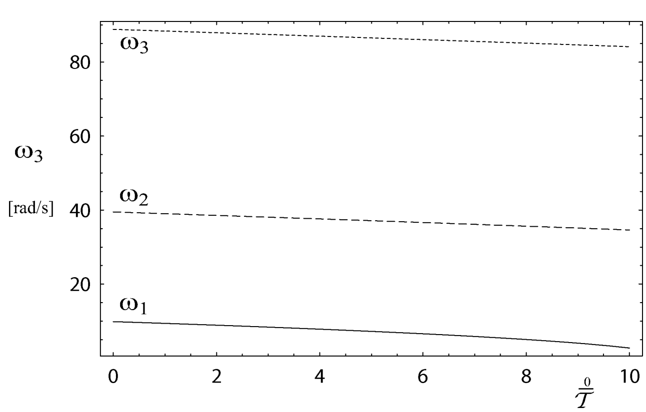

Fig. 2 Effect of the temperature on the first three eigenfrequencies, $\delta_e = 0$ and $\delta_\alpha = 0$.

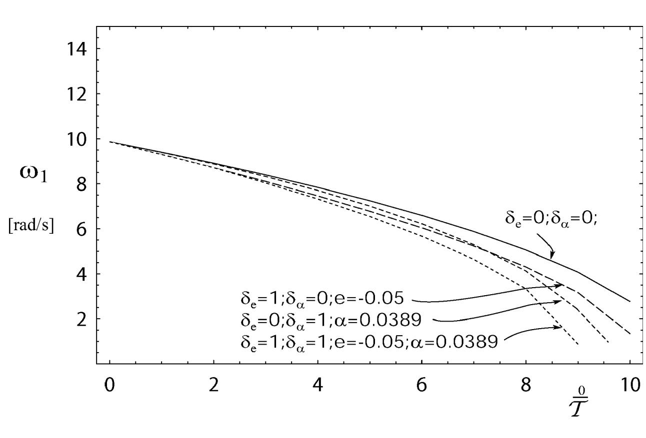

Fig. 3 Effect of the temperature and the thermal degradation on the first eigenfrequency.

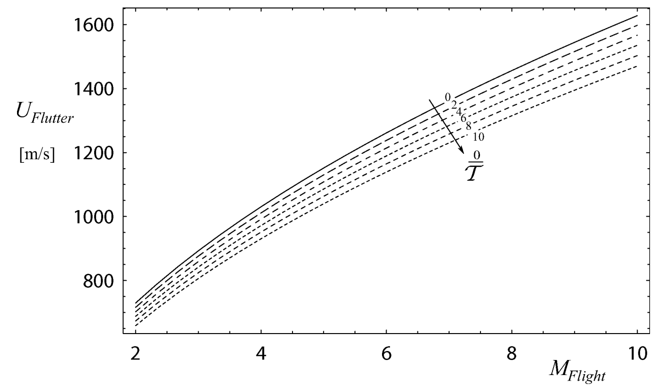

Fig. 4 Flutter speed vs Mach flight; effect of the thermal field, no thermal degradation, $\delta_e = 0$ and $\delta_\alpha = 0$.

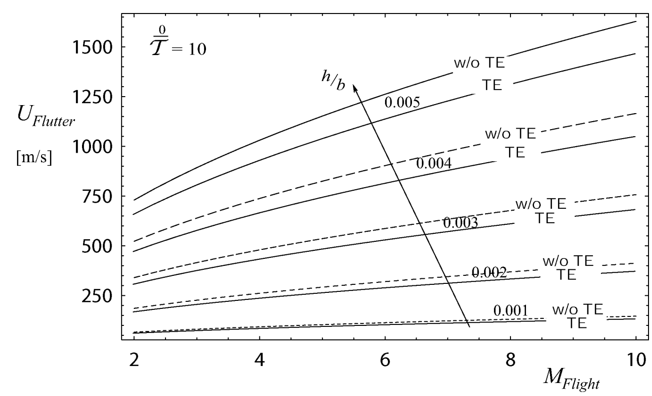

Fig. 5 Effect of the thickness ratio h/b on the flutter speed vs Mach flight with and without presence of thermal field [with and without thermal effect (TE)]; no thermal degradation, $\delta_e = 0$ and $\delta_\alpha = 0$.

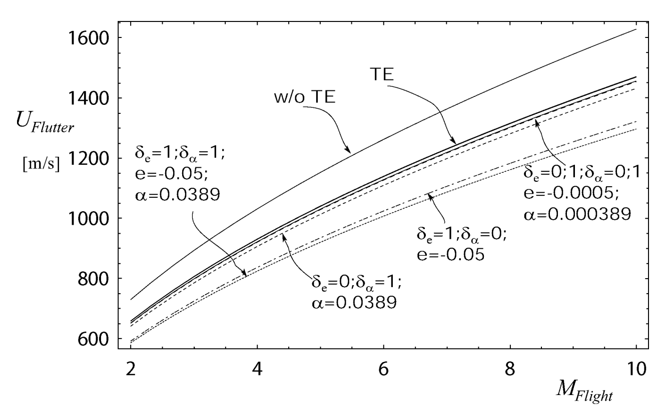

Fig. 6 Flutter speed vs Mach flight; effect of the thermal field ($\overset{0}{\mathcal{T}} = 10$) with thermal degradation.

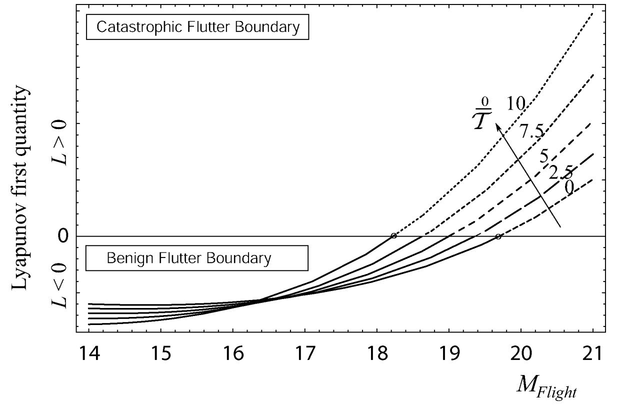

Fig. 7 Benign and catastrophic portions of the flutter boundary; effect of thermal field in presence of structural and aerodynamic nonlinearities, no thermal degradation, $\delta_e = 1$ and $\delta_\alpha = 0$.

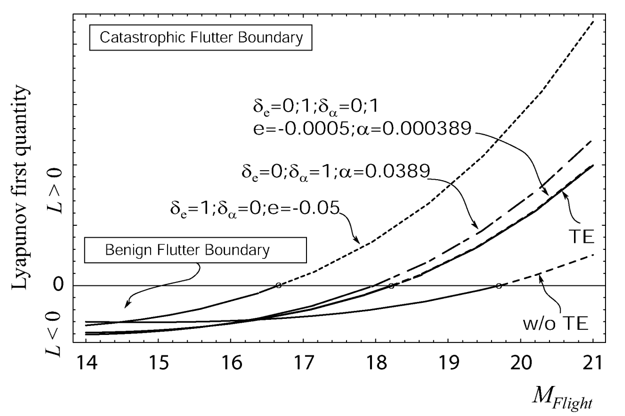

Fig. 8 Benign and catastrophic portions of the flutter boundary; effect of thermal field ($\overset{0}{T} = 10$) with thermal degradation in presence of structural and aerodynamic nonlinearities.

the increase of the thermal field, a shift of the transition between the benign and catastrophic flutter boundary toward smaller values of the flight speed is occurring. This reveals that the temperature exerts a detrimental effect not only on the flutter boundary but on the character of the flutter boundary as well. It also clearly appears that the aerodynamic nonlinearities are, in general, destabilizing.

Figure 8 shows the effect of the thermal degradation on the character of the flutter boundary. Also in this case, the effect of the thermal degradation on the elastic modulus is prevalent, and the occurrence of the catastrophic flutter is shifted toward smaller values of the flight speed.

Conclusions

A number of results and conclusions related to the supersonic flutter of infinitely long thin-walled flat panels operating is a thermal field have been presented. In this context, the implications of structural and aerodynamic nonlinearities in conjunction with the temperature field and the thermal degradation, on the character, benign or catastrophic, of the flutter critical boundary, have been examined. It was also shown that, at high flight Mach numbers, the aerodynamic nonlinearities contribute invariably to the catastrophic character of the flutter boundary. This implies that, with the increase of the supersonic/hypersonic flight speed, when the aerodynamic nonlinearities become prevalent, the flutter boundary becomes catastrophic, irrespective of the presence of structural nonlinearities. It was also shown that the effect of temperature and thermal degradation are invariably detrimental in the sense of reducing the flutter speed and of rendering the flutter boundary a catastrophic one. In addition, as a byproduct of this analysis, conclusions on the effects of the temperature field coupled with those of the thermal degradation on the eigenfrequency and flutter boundary have been outlined. Finally, the concept of LFQ used here enables one to derive results related to stability/instability of LCO in a more general context than within the usual procedures based on computational methodologies.

Appendix A: Dimensionless Parameters

$$\bar{w} = \frac{w}{h}, \qquad \bar{y} = \frac{y}{b}, \qquad \bar{t} = \frac{t}{\tau}$$

$$\bar{\rho} = \frac{\rho_\infty b}{\rho_p h}, \qquad \overset{0}{\mathcal{T}} = \frac{\overset{0}{T}}{\hat{T}}, \qquad \tau = \sqrt{\frac{b^4 \rho_p h}{D_0}}$$

$$\overset{0}{T} = \overset{0}{\mathcal{T}} \sin(\pi \bar{y}), \qquad \hat{T} = \frac{D_0}{\left(E_0 h b^2 \alpha_0\right)}, \qquad \lambda = \frac{2 q_\infty b^3}{M_\infty D_0}$$

$$D_0 = \frac{E_0 h^3}{12(1-\mu^2)}, \qquad q_\infty = \frac{\rho_\infty U_\infty^2}{2}$$

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